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which is the required formula.<sup>1</sup>

(b) Since  $a$  and  $d$  are positive acute angles, the last term cannot become negative and is zero only when  $x = 0$ . Therefore  $d$  will have a minimum value when  $x = 0$ , so that  $r = r' = \frac{1}{2}a$ , and  $i = i'$ , and the complete ray is symmetrically situated with respect to the prism.

(c) For this minimum value of  $d$ , say  $d_0$ , we have the classical laboratory formula

$$n = \frac{\sin \frac{1}{2}(a + d_0)}{\sin \frac{1}{2}a}.$$

**2879 [1921, 89]. Proposed by E. J. OGLESBY, Washington Square College.**

Given the values of  $U_{5:9}$ ,  $U_{5:10}$ ,  $U_{5:11}$ ,  $U_{6:9}$ ,  $U_{6:10}$ ,  $U_{6:11}$ ,  $U_{7:9}$ ,  $U_{7:10}$ ,  $U_{7:11}$  where  $U_{h:k} = \sqrt{hk}$ , find the value of  $U_{6.2:9.3}$  by interpolation.

SOLUTION BY J. B. REYNOLDS, Lehigh University.

From the first three given terms we find by the method of differences<sup>2</sup>  $U_{5:9.3} = 6.8190$ , from the next three  $U_{6:9.3} = 7.4698$  and from the last three  $U_{7:9.3} = 8.0635$ . Using these three values then as a series by the same method we find  $U_{6.2:9.3} = 7.5937$ .

Also solved by H. N. CARLETON.

**2881 [1921, 89]. Proposed by E. B. ESCOTT, Oak Park, Ill.**

If, in the polynomial  $X^3 - 2$ , we substitute  $x^2 + x - 4$  for  $X$ , the given expression can be factored, that is,  $X^3 - 2 \equiv (x^3 + 3x^2 - 3x - 11)(x^3 - 6x + 6)$ . Find a substitution for  $X$  so that the polynomial  $X^3 + pX^2 + qX + r$  may be factored.

SOLUTION BY THE PROPOSER.

Let

$$X^3 + pX^2 + qX + r = (X - b)(X + c)^2 - a^2(X - d)^2. \quad (1)$$

Expanding the second member and equating coefficients of like powers of  $X$ , we get

$$a^2 + b - 2c + p = 0, \quad 2a^2d - 2bc + c^2 - q = 0, \quad \text{and} \quad a^2d^2 + bc^2 + r = 0.$$

Eliminating  $b$  and  $d$  and solving for  $a^2$  we get

$$a^2 = \frac{(3c^2 - 2pc + q)^2}{4(c^3 - pc^2 + qc - r)}.$$

Therefore, it is necessary and sufficient that<sup>3</sup>

$$c^3 - pc^2 + qc - r = n^2,$$

and we get by substitution

$$a = \frac{3c^2 - 2pc + q}{2n}, \quad d = -\frac{c^3 - qc + 2r}{3c^2 - 2pc + q}, \quad \text{and} \quad b = -a^2 + 2c - p.$$

<sup>1</sup> We might say

$$\begin{aligned} \frac{1}{n^2} &= \frac{\sin^2 \frac{1}{2}a \cos^2 x}{\sin^2 \frac{1}{2}(a + d)} + \frac{\cos^2 \frac{1}{2}a \sin^2 x}{\cos^2 \frac{1}{2}(a + d)} \\ &= \frac{\sin^2 \frac{1}{2}a}{\sin^2 \frac{1}{2}(a + d)} + \left[ \frac{\cos^2 \frac{1}{2}a}{\cos^2 \frac{1}{2}(a + d)} - \frac{\sin^2 \frac{1}{2}a}{\sin^2 \frac{1}{2}(a + d)} \right] \sin^2 x \\ &= \frac{\sin^2 \frac{1}{2}a}{\sin^2 \frac{1}{2}(a + d)} + \frac{4 \sin \frac{1}{2}d \sin(a + \frac{1}{2}d) \sin^2 x}{\sin^2(a + d)} \end{aligned}$$

and use this formula for (b) and (c) instead of the formula in the text,  $d$  being a minimum when  $[\sin^2 \frac{1}{2}a]/[\sin^2 \frac{1}{2}(a + d)]$  is a maximum.—EDITORS.

<sup>2</sup> See 1921, §30.

<sup>3</sup> Thus it seems to be a necessary part of the hypothesis that there is a rational number  $c$  that will make the given polynomial equal to minus the square of a rational number.

Letting  $f$  denote the polynomial, if  $f(-c) = -n^2$  we can write

$$f(X) = (X + c)^2 \left[ X - 2c + p + \left( \frac{f'(-c)}{2n} \right)^2 \right] - \left[ \frac{f'(-c)}{2n} (X + c) - n \right]^2,$$

which we can make the difference of two squares by putting  $X - 2c + p + \left( \frac{f'(-c)}{2n} \right)^2 = (x + t)^2$ .

—EDITORS.